Introduction to Mathematical Models

of the Epidemiology & Control of Infectious Diseases

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Project 1A: 2014 Ebola outbreak in Freetown (Sierra Leone)

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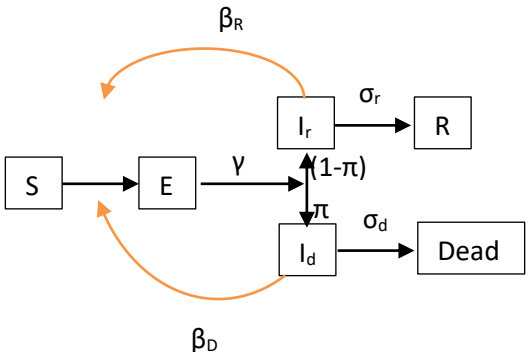
Examples of models

Below you will find some flow charts corresponding to 3 examples of models coded in the Odin interface (see shared solutions). Those models are illustrative of what could be done alongside the data analyses as part of the practical. In the saved versions of the models, some parameters have been fitted using the "Fit model" button in the 'Fit' tab.

Model 1:

This is a simple model with change in reproduction associated with the time of intervention.



Variables

S: susceptible, E: exposed, I_r and I_d: infectious who will eventually recover or die respectively, R: recovered.

Parameters

$\beta_{D/R}$: transmission rates, in this model two values exist associated with infectious cases who will either die or recover. In the parameterisation (see Odin interface solutions), it is assumed that the overall infectiousness of dying or recovering cases are the same but distributed along a shorter period for the dying case as the delay from onset to final outcome is longer (therefore they are more infectious per unit time). Thus the individual basic reproduction number of cases who will eventually recover or die is the same.

π: Case Fatality Ratio (CFR), $1/\gamma$: incubation period, $1/\sigma_r$ and $1/\sigma_d$: delay to recover or dying respectively.

Impact of the intervention

On day 170, the effective reproduction number is decreased, therefore the rate of transmission also decreases.

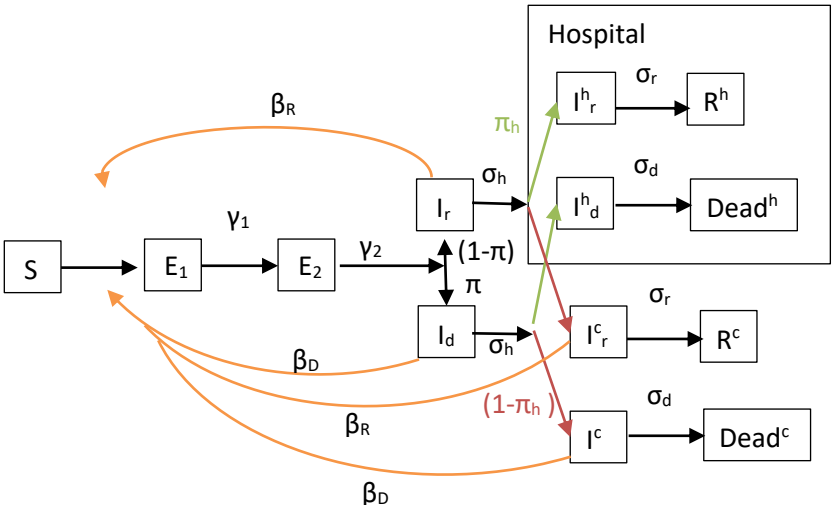
Model 2:

This model increases complexity by:

- Accounting for the non exponential distribution nature of the incubation period,
- Explicitly modelling hospitalisation (and therefore those who stay in the community).

In perhaps the most important feature of this model, it is assumed that cases in hospital are not transmitting the virus further, i.e. they are perfectly isolated.

The intervention is then modelled as a decrease in the delay to isolate cases.



New variables

E_1 and E_2: two exposed classes (to approximate the observed Gamma distribution of incubation period). Others variables are as before with 'h' subscript and 'c' subscript indicating cases in hospital or community respectively.

New parameters

$1/\gamma_1+1/\gamma_2$: mean incubation period, π_h proportion of cases who seek health care, $1/\sigma_h$ delay between onset and hospitalisation.

Note that the rates of infection are defined from a constant basic reproduction number here (i.e. no changes before/after the intervention), which is defined for the cases that remain in the community.

Impact of the intervention

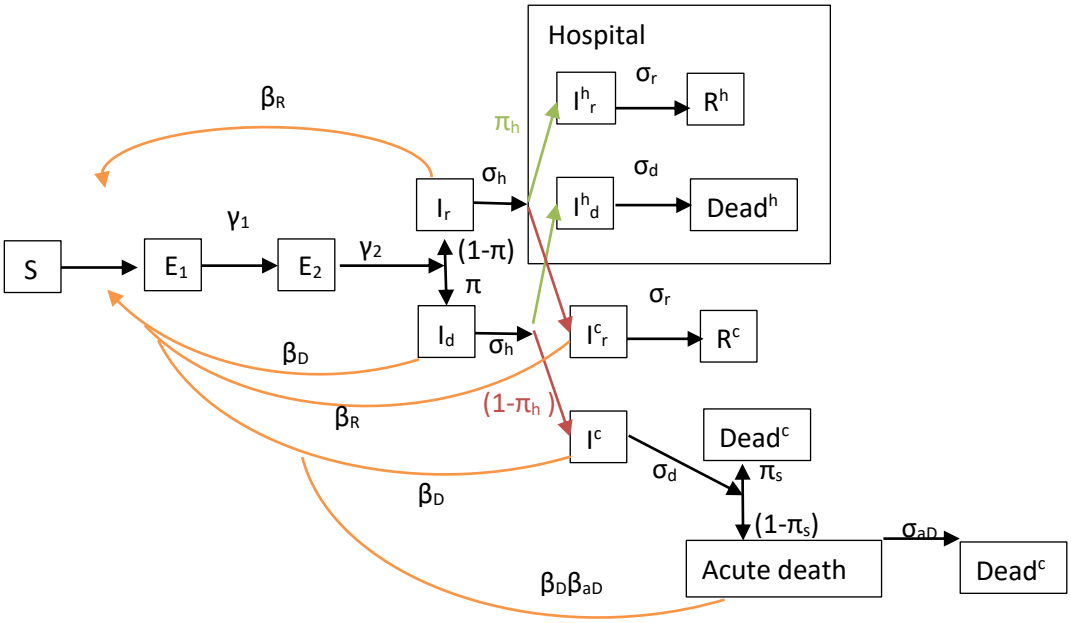
On day 170, the delay between onset of symptoms and hospitalisation ($1/\sigma_h$) is decreased. This induces a reduction in the effective reproduction number as cases in hospital are assumed to be isolated and therefore do not contribute to onward transmission.

Model 3:

This model further increases complexity by:

- Accounting for the heightened infectiousness that occurs during unsafe burials.

In addition to modelling a change in the delay to isolate cases (as in mode 2), this model allows for the proportion of unsafe burial to be reduced.



New variables

Acute death: for cases who will eventually die in the community, some are not buried safely ($1-\pi_s$) and for those, infectiousness continues after death (for $1/\sigma_{aD}$). The transmission potential during this stage is known to be important.

New parameters

$1/\sigma_{aD}$: duration of the acute death stage,

π_s proportion of cases that die and are safely buried.

β_{aD}: the factor of increased infectiousness during the unsafe burial, i.e. the transmission rate during the funeral is $\beta_D\beta_{aD}$.

Again a single basic reproduction characterises transmissibility and is constant over time. Such basic reproduction number apply to cases who recover while staying in the community or those who die while staying in the community and for whom death is followed by an unsafe burial.

Accordingly, hospitalised cases have a lower individual reproduction number (i.e. they are isolated from the time of their hospitalisation, and burials in hospital are assumed to be always safely performed). Additionally, among those staying in the community, the individual reproduction number among those safely buried is also lower.

Finally in the model presented, it is assumed that, for those who died in the community and were unsafely buried, half of the transmission occurred during the burial.

Impact of the intervention

On day 170, the delay between onset of symptoms and hospitalisation ($1/\sigma_h$) is decreased as before, and the proportion of safe burials is also increased. This induces a reduction in the effective reproduction number as:

- cases in hospital are assumed to be isolated and therefore do not contribute to onward transmission
- safe burial procedure reduces the risk of onward transmission for those who die in the community.